SMH modelling

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Remember this?

$$\frac{d^2x}{dt^2} = -\frac{kx}{m}$$

We can model the motion of a spring/mass shm oscillator using this as well as with the solutions that we saw with sin/cos in them. This is quite simple, but requires a lot of concentration and repetition - therefore, it is often done via a spreadsheet.

The method is knwo as the 'Iterative' approach - or 'iteration' - or the 'frustratingly fiddly method that usually goes wrong'.

It is best worked out with an example.

A 0.5kg mass is attached to a spring of k=10N/m. The mass is displaced by 0.05m then allowed to oscillate freely. What is its displacement after 0.2 seconds. Use an iterative method with an interval of 0.1s.

You could just use a formula... But this time here's the iterative method.

Recall

- $\frac{d^2x}{dt^2} = -\frac{kx}{m}$. Also $a = \frac{dv}{dt}$ so $\frac{dv}{dt} = -\frac{kx}{m}$
- Now we are going to use fixed time intervals

  we can rewrite this as $\frac{\Delta v}{\Delta t} = -\frac{kx}{m}$

  so $\Delta v = -\frac{kx}{m}\Delta t$ .... (1)
- we also know $v = \frac{\Delta x}{\Delta t} \Rightarrow \Delta x = v\Delta t$ ... (2)

Create a table:

t/s	$\Delta v/ms^{-1}$	v/ms^{-1}	$\Delta x/m$	x/m

Fill in the initial conditions:

t/s	$\Delta v/ms^{-1}$	v/ms^{-1}	$\Delta x/m$	x/m
0		0		0.05

Now consider the first conditions at the end of the first interval. Ie t=0.1

$\Delta v = -\frac{k}{m}x\Delta t$ so $\Delta v = -\frac{10}{0.5} \times 0.05 \times 0.1 = -0.1$

The new V is "the previous V" + the change over the interval.

Add this to the table.

t/s	Δv/ms^{-1}	v/ms^{-1}	Δx/m	x/m
0		0		0.05
0.1	−0.1	−0.1		

Next we can find Δx by $\Delta x = v\Delta t = -0.1 \times 0.1 = -0.01$

The value of x is $\therefore$ previous value + $\Delta x = 0.05 + -0.01$
$= 0.04$

t/s	Δv/ms^{-1}	v/ms^{-1}	Δx/m	x/m
0		0		0.05
0.1	−0.1	−0.1	−0.01	0.04

Now its repeat the previous steps. I am going to do them fully here rather than assume its easy. Because it isn't that easy!

Next t is 0.2.

Again $\Delta v = -\frac{k}{m} x\Delta t \Rightarrow \Delta v = -\frac{10}{0.5} \times 0.04 \times 0.1 = \underline{-0.08}$

v at the end of second interval = previous value + new Δv
$= -0.1 + -0.08 = \underline{-0.18}$

Now $\Delta x = v\Delta t \Rightarrow \Delta x = -0.18 \times 0.1 = -0.018$

So x (after 2nd interval) = prev value + new Δx
$= 0.04 + -0.018$
$= 0.022$

Add these new values:

t/s	Δv/ms^{-1}	v/ms^{-1}	Δx/m	x/m
0		0		0.05
0.1	−0.1	−0.1	−0.01	0.04
0.2	−0.08	−0.18	−0.018	0.022

Ta da!

Of course there are problems with this method:

- Its prone to errors
- It assumes that velocity doesn't change over the interval (or accel, or x)

This last point means that the iterative method gives us approximate answers. You can improve the accuracy by decreasing the value of the time interval. This is only partially helpful, since you have a pile more calculations meaning more likelihood of error. Clearly, this is where spreadsheets come into their own.

Now you are aware of the method take a moment to consider just what inspiring people the 'old scientists' were - no spreadsheets or fancy measuring methods - or even calculators yet they worked out all these eqns and carefully used the iterative method.

Max respect is due.

Apparently this chap, Gauss, was the first person to use it around 1777 (though I find that hard to believe....)

https://en.wikipedia.org/wiki/Iterative_method